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A FLUID MODEL OF THE CYCLIC SERVICE SYSTEM

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The rapid development of the service sector in many economies has stimulated a renewed interest in multi-server queueing models with abandonment and re-entry. These models are particularly important in large-scale service systems, such as customer contact centers and health-care centers. At present, queueing theory is held to be the principal mathematical tool to describe processes in waiting lines using the apparatus of probability theory. For a broad range of purposes, however, deterministic, so-called “fluid”, models originally intended to analyze the stability of an underlying queueing system, prove to be an effective alternative to discrete stochastic queueing models. They are an approximation technique for studying important dynamic characteristics of queueing networks.

Our research fits into the single customer class, single server type category of queueing systems with multiple servers, unlimited-size buffer, abandonment and re-entry. The novelty of our approach is an idea that rendering a service to a customer is cybernetically identical to the conversion of a substrate into a different molecule in an enzyme-catalyzed biochemical reaction. Indeed, service is a process of interaction between two economic entities, provider and customer, which changes the condition of the customer by creating a new value for him/her. The “fluid” model we propose is the system of nonlinear ODEs:

$$\begin{aligned}
\dot{x} &= r + k_{-1}v + k_3 y - k_1 ux - k_4 x,\\
\dot{y} &= k_2 v - (k_3 + k_5)y,\\
\dot{u} &= (k_{-1} + k_2)v - k_1 ux,\\
\dot{v} &= k_1 ux - (k_{-1} + k_2)v.
\end{aligned}$$

Here x is the number of the potential customers waiting for service, y is the number of successfully served customers, u is the number of idle servers, v is the number of busy servers, r is the arrival rate of the customers, k_1 is the capture rate of a would-be customer by a server, $1/k_{-1}$ is the mean time it takes for the order to become withdrawn or denied once customer-server contact is initiated, k_2 is server's turnover number, or the maximum number of fulfilled orders that a single server is capable to deliver at one unit of time, so that $1/(k_{-1} + k_2)$ is the mean processing time of a server (service time), $1/k_3$ is the lifetime of a commodity obtained at a provider, k_4 is the specific rate with which the impatient potential customers leave the queue, and k_5 is the specific out-migration rate of the past customers.

The parametric analysis of the model's steady-state is carried out, particularly, how the arrival rate of new customers affects the queue length. It is also shown that the model implies a saturating response of the yield to the work-in-progress providing the service time is much shorter than the waiting time.